

AN APPROXIMATE LIGHT CONE METHOD TO INVESTIGATE MESON STRUCTURE

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We present results obtained using a transverse lattice formulation of QCD to calculate light and heavy meson observables.

1 Introduction

It is well-established that the light-front formulation of QCD make calculations of observables in high-energy phenomena like DIS considerably easier while providing a more natural interpretation of the same ¹. Transverse lattice formulation of QCD attempts to exploit the benefits ^{2,3,4} from using the LF gauge and the well-known advantages of the more conventional Euclidean and Hamiltonian lattice formulations.

Transverse lattice techniques were employed quite successfully to pure glue calculations ^{5,6}. Encouraged by these results, the *femtoworm approximation*, which we describe briefly in the next section, was conceived to include the fermions on the transverse lattice, as well. ^{7,8,9,10,11}

In this paper, we briefly describe the femtoworm approximation in section 2, referring the reader to relevant references for the most part. Some typical results and conclusions are included in section 3.

2 The Hamiltonian of the Femtoworm Approximation

Using light cone gauge requires that the longitudinal light cone space, x^+ and light cone time, x^- variables be redefined in terms of the equal time longitudinal space, x^0 and time, x^3 coordinates as $x^\pm = x^0 \pm x^3$. The transverse directions are the same as in equal time definitions but discretized. This enables the use of a gauge invariant regulator in the theory while simultaneously retaining boost invariance in the longitudinal direction. In addition, the femtoworm approximation is based on the following considerations:

- We work in the large N_c limit.
- A light cone Tamm-Dancoff truncation is applied so that the Fock state expansion of the meson states include only 2-particle and 3-particle Fock components.

- One link approximation: The maximum number of link fields that can separate a q and an \bar{q} is one.
- Interactions between 2-particle to 3-particle states (and vice versa) can take place only through two interaction terms of the Hamiltonian – one that flips the spin of the interacting fermions and the other that does not.

The 2-particle states consist of a $q\bar{q}$ pair sitting on the same \perp lattice site. The 3-particle states consist of a q and an \bar{q} occupying neighboring sites separated by a link field (the gauge degrees of freedom in this approach). The 2-sector part of the Hamiltonian contains two terms that describe the kinetic energies of the q and the \bar{q} , respectively. It also contains a longitudinally confining Coulomb interaction term acting between the pair. The 3-sector part of the Hamiltonian contains three terms that describe the kinetic energies of the q , the \bar{q} and the link field, respectively. There are two more terms – one describes a longitudinally confining Coulomb interaction between the q and the link field at the site where the q resides and another that describes a longitudinally confining Coulomb interaction between the \bar{q} and the link field at the neighboring site where the \bar{q} resides. Note that there are no Coulomb terms connecting the q and the \bar{q} on neighboring sites. This is because these interaction terms are local on the \perp lattice sites. Further, such interactions would yield non-planar diagrams which would violate our large N_c approximation.

Transverse propagation of the mesons take place through the two interaction terms that connect the two Fock states. The meson propagates by emitting (2- to 3-sector) or absorbing (3- to 2-sector) a link field. When a meson is in the 2-sector (i.e., q and \bar{q} are sitting on the same \perp lattice site), one of it's constituents can hop to a neighboring site emitting a link field either through the spin flip term in which case it's spin will flip or through the spin non-flip term in which case it's spin will not flip. The meson in this configuration (a q and an \bar{q} on neighboring sites separated by one link field) represents it's 3-particle state. When the meson is in this state, one of the fermion degrees of freedom hops and joins the other fermion on a neighboring site by absorbing the link field either through the spin flip in which case it's spin will flip or through the spin non-flip interaction term in which case it's spin will not flip. The nature of the propagation is reminiscent of that of an inchworm on a scale of a femtometer which motivated the name *femtoworm*.

The strength of spin flip and spin non-flip interaction terms are labeled m_v and m_r , respectively. Though, in general, a link field emitted through a spin flip (spin non-flip) interaction term can be absorbed through a spin non-flip (spin flip) interaction term, such an interference term is disallowed in the $\mathbf{k}_\perp = \mathbf{0}$ case. This and other spin dynamics are discussed in details in ^{7,9}.

The parameter space that defines the above $q\bar{q}$ bound state Hamiltonian contain the following parameters:

- kinetic masses for the light quark in the 2 and 3 particle Fock sector.
- kinetic masses for the strange quark in the 2 and 3 particle Fock sector.
- kinetic mass for the link field.
- spin flip and non-flip hopping terms for the light quark.
- spin flip and non-flip hopping terms for the strange quark and
- longitudinal gauge coupling.

As explained in ⁹, this parameter space is constrained by:

- **imposing Lorentz symmetry.** This is done by demanding that the eigenstates of the Hamiltonian have the same dispersion relation.
- **imposing chiral symmetry in the spontaneously broken form.** This is implemented by tuning the spin flip and spin non-slip hopping terms to yield the observed $\pi - \rho$ mass splitting and
- **fixing the physical string tension.** We used the string tension as obtained from glueball calculations ⁶.

3 Results and Conclusions

We have studied the LF momentum distribution amplitudes of several light mesons (π, ρ), mesons with strange quarks (KK^* system, ϕ) as well as heavy mesons (B -meson). For each of the above, we have also calculated their respective parton distribution functions.

The LF momentum distribution of the pion [Fig.1(a)] is quite close to the asymptotic shape. The first five even coefficients of a Gegenbauer projection of the momentum distribution, $\psi(x) = 4x(1-x) \sum_{i=0}^{\infty} a_i G_i^{\frac{3}{2}}(1-2x)$, are $\{1.5, 0.102, -0.006, -0.005, -0.001\}$. The tail of the parton distribution function [Fig.1(b)] is somewhat higher than what is observed. Because there is only one link, most of the longitudinal momenta close to $x = 1$ are carried by the fermions.

The dashed line in Fig.1(c) shows the \sqrt{z} behavior of the B -meson LF momentum distribution in the heavy quark limit. For phenomenological applications ¹³ it is useful to have numerical estimates for the moments ¹⁰ (the normalization is $\int_0^1 dz \phi_{\infty}(z) dz = 1$) for which we find $\int dz z \phi_{\infty}(z) = 1.51 GeV$ and $\int dz z^{-1} \phi_{\infty}(z) = 1.22 (GeV)^{-1}$. For the decay constant ¹⁰, which plays also an important role in mixing phenomenology, we find $f_B \approx 240 MeV \pm 20 MeV$. Fig. 2 shows the Isgur-Wise form factor of the B -meson in the heavy quark limit which allows one to extract the V_{cb} element of the CKM matrix from the zero recoil point of a semi-leptonic decay channels like $B \rightarrow \bar{D}^* l \nu$. We obtain values of V_{cb} in the range $0.0415 - 0.0420$

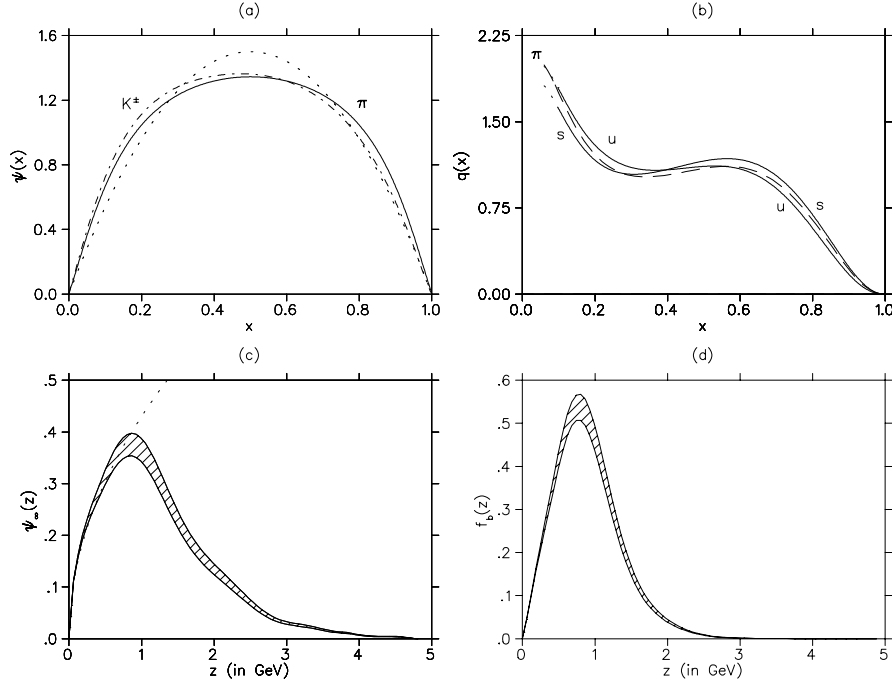


Figure 1. (a) LF momentum distribution amplitude of the π (bold), K^\pm (dashed) and the asymptotic shape (dotted). (b) Parton distribution function of the u -quark in π (dashed), the u and s distributions in K^\pm . (c) B -meson distribution amplitude in the heavy quark limit. (d) B -meson LF momentum distribution in the heavy quark limit. Systematic uncertainties arising from the numerical extrapolation are indicated by the shaded area.

which is very consistent with the experimental data ¹².

Since we used basis function techniques and extrapolated the results in Hilbert space dimensions, uncertainties from Hilbert space truncation are under control. Systematic errors arising from the Fock space truncation cannot be estimated unless higher Fock components are included. Thus, a main improvement over this work would be to include such Fock components. For example, we expect that including such components will significantly reduce the 2-particle Fock space amplitude, thereby reducing the decay constants which are calculated ^{9,10} from the normalizations of the LF distribution amplitudes. Other observables are expected to be effected in a similar manner.

For the heavy quark observables, $1/M_b$ and perturbative corrections are

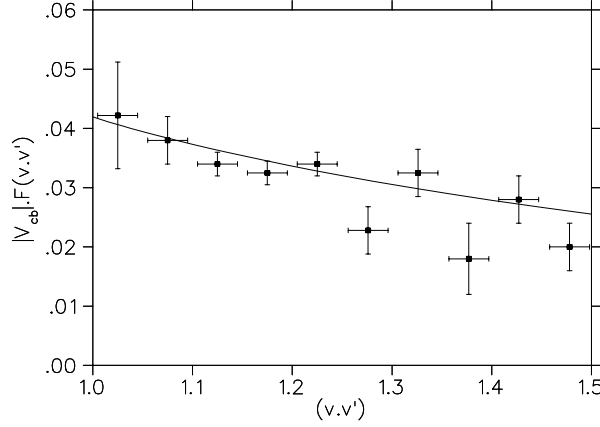


Figure 2. Isgur-Wise form factor in the heavy quark limit. Uncertainties from the $M_b \rightarrow \infty$ limit were very small for this observable. The experimental data is from Ref. [12] .

expected to contribute at zero recoil ($v.v' = 0$). We were not able to estimate the former because we worked in the $M_b \rightarrow \infty$ limit. Also, it is not known how to reach small lattice spacings on the \perp lattice and as such an estimate of the latter corrections are beyond the scope of this technique. Despite this caveat, we feel that with further improvements, this technique holds promise.

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